B.Sc.–I 1st Semester (Hons.) Examination,

November–2014

MATHS

Paper–BHM–111

Algebra

Time allowed : 3 hours] *[Maximum marks : 60*

Note : *Attempt **five** questions, selecting **one** question from each section. **Question No. 9 is compulsory.***

Unit–I

1. (a) Show that all +ve odd integral powers of a skew symmetric are skew symmetric while +ve even integral powers are symmetric.

(b) Reduce the matrix $A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ to column echelon form and hence determine its column matrix.

2. (a) Determine the values of a, b and c so that (1, 0, –1) and (0, 1, –1) are eigen vectors of the

matrix $\begin{bmatrix} 2 & 1 & 1 \\ a & 3 & 2 \\ 3 & b & c \end{bmatrix}$.

(b) Find the characteristic equation and minimal

equation of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$.

Unit–II

3. (a) Find the values of a and b for which the following system of linear equations

$2x + by - z = 3,\ 5x + 7y + z = 7,\ ax + y + 3z = a$

has an infinite number of solutions.

(b) Prove that the inverse of a unitary matrix is unitary and its determinant is equal to unity.

4. (a) Reduce the quadratic form

$x^2_1 + 2x^2_2 + 3x^2_3 - 4x_2 x_3 + 6x_3 x_1$ to canonical form and find the rank, index and signature of the form. Also find the equations of linear transformations.

(b) Determine the definiteness of the following quadratic form in R^3 with the help of leading principal minors

$$6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1 x_2 - 2x_2 x_3 + 4x_3 x_1$$

Unit–III

5. (a) Solve the equation

$$2x^6 - 3x^5 + 5x^4 + 6x^3 - 27x + 81 = 0$$

whose one root is $\sqrt{2} + i$

(b) Solve the equation $4x^4 - 4x^3 - 25x^2 + x + 60$ given that the difference between two roots is unity.

6. (a) Solve the equation $15x^4 - 8x^3 - 14x^2 + 8x - 1 = 0$, given that the roots are in H.P.

(b) If α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$ form an equation whose roots are $\frac{\beta+\gamma}{\alpha}, \frac{\gamma+\alpha}{\beta}, \frac{\alpha+\beta}{\gamma}$.

Unit–IV

7. (a) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method.

(b) Solve by the method of resolution into quadratic factors $x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$

8. (a) Solve $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ by Ferrari's method.

(b) Show that the equation $2x^7 + 3x^4 + 3x + k = 0$ has at least 4 imaginary roots for all values of K (Constant).

Unit–V

9. (a) If A is skew Hermitian, show that iA is Hermitian.

(b) Find the eigen value of A^{-1} where $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

(c) Define orthogonal matrix and give example.

(d) If α, β, γ are roots of the equation $x^3 + qx + r = 0$ then find $\sum \frac{1}{\alpha}$

(e) Find the condition that the roots of the equation $x^3 + 3px^2 + 3qx + r = 0$ are in A.P.

(f) Discuss the nature of roots of the equation $x^3 + 2x + 1 = 0$.